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CERTAIN MODELS OF MILLING PROCESS USED FOR ASSESSING OF DYNAMIC STABILITY SYSTEM OF MACHINE TOOLS

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ABSTRACT: This paper is presented some models of milling process used for evaluation of dynamic stability of machine tools. In addition, a model of milling process by used a side-milling cutter with inserts in details is presented. These mathematical models together with experiments test assure the investigation of dynamic stability system of machine tools and lead to enhance the performances of process.

1. INTRODUCTION

During of cutting process the strong vibrations can occurred, which affect the accuracy of machining, tool wear, etc. These perturbations maybe caused by different items: thickness of cut width of cut, spindle speed, rake angle, effect-parameters of cutting force, effectiveness thickness cut, temperature in contact zone, etc. [1-17].
A dynamic machining system (DMS) is composed by an elastic system (SE)-formed by workpiece-tool subsystem, with one DOF (degree of freedom) as Kelvin-Voight associated model, and the cutting system (PA) by the static stiffness-K_s and cutting force-F, in Fig.1 is presented [4-6].

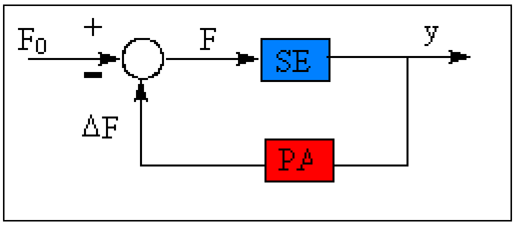

Fig.1. Diagram of DMS with one-DOF [4-6]

$$m\ddot{y} + c\dot{y} + Ky = F$$ (1), which [4-6,19]:

$$F = F_o - \Delta F, \qquad \Delta F = K_s\left(1 - \mu e^{-pT_a}\right)\cdot y \qquad (2),$$

where: ΔF-dynamic force, T_a = decay time, μ = overlap factor. In final Eq. (1) becomes:

$$m\ddot{y} + c\dot{y} + \left[K + K_s\left(1 - \mu e^{-pT_a}\right)\right]y = F_o \qquad (3)$$

During of cutting process can occur three relative trajectories of the point on the cutting edge of cutter: predictable trajectory (as reference), unperturbed trajectory (get by F_o), and perturbed trajectory (get by variation of thickness cut-*a*, etc.) [4-6].
The DMS is stable when it has the capacity to maintain too closed the perturbed trajectory near unperturbed trajectory, and unstable system in otherwise. If μ=0, and $K + K_s = \overline{K}$, the Eq.(3) becomes (F_0=0):

$$m\ddot{y} + c\dot{y} + \overline{K}y = 0 \qquad (4)$$

The mathematical model presented in Eq. (4) is stable if its answer-y_1 is canceled in time, which means [4-6]:

$$\lim_{t\to\infty} y_1 = 0$$ (5), and unstable if:

$$\lim_{t\to\infty} y_1 \to \infty \tag{6}$$

For the linear DMS (μ≠0) a main importance is presented by the absolute stability. The SDP with delay is absolute stable if its answer is asymptotic stable. For some values of SDP's parameters, the system can be stable, and by other values is unstable.
The DMS parameters range with zero entry, on which occurs a transient system with decay in time and any initial condition of motion, it's named the stability domain, and for system that has arise in time of transient answer is named absolute unstable domain. At stability border of these systems is a DMS' answer of approximately harmonic shape and its frequency is near of the eigenfrequency's structure.

2. MODELS OF MILLING PROCESS

For many years, the scientific study of researchers has been focused of development of many models of milling process, which assessing the stability of machine tools and milling processes [1-9, 20-28].
Many researchers have used Nyquist techniques [10,21-26] in an iterative manner to generate stability lobs diagrams (SLDs) by Sridhar et al. (1968), Minis and Yanushevsky (1990, 1993), Lee and Liu (1991). Time domain simulation (TDS) [3,22-28] is another technique that can be use to generate SLDs by Tlusty and Ismail (1981, 1983), Tlusty (1986), Tsai et al. (1990), Smith and Tlusty (1993), Tarng et al. (1994), Weck et al. (1994). Another technique analyzing chatter that generates analytical SLDs [1-3] was been introduced by Budak and Altintas (1995), Altintas (2000). Spindle speed variation (SSV) [2,7-9,20] is another technique for chatter suppression, by Lin et al. (1990), Radulescu et al. (1997), Altintas et al. (1998). Chatter analysis with nonlinear force processes [7,24-29] was developed by Zhang and Ni (1995), Endres (1996), Landers and Ulsoy (1996, 1998, 2002).
For the beginner, we'll start with a model of milling cutting process at orthogonal process by used a cylindrical milling cutter with straight teeth, presented in Fig.2 [4,6].

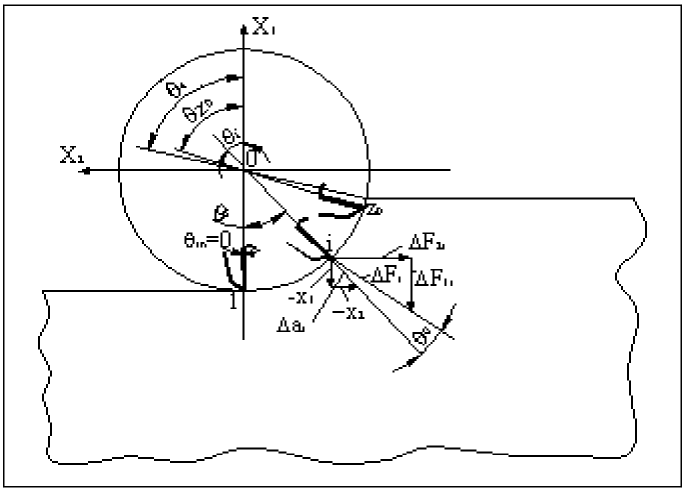

Fig.2. The model of orthogonal milling process used milling cutter with straight teeth [6].

By noted of $k_i^{(a)}$-the stiffness of cutting force for one tooth, the dynamic variation of cutting force is [4,6,19]:

$\Delta F_i = k_i^{(a)} \cdot \Delta a_i = -k_i^{(a)}\left(1-\mu e^{-pT}\right)\left[\cos\theta_i, \sin\theta_i\right]x$ (7), where: $x=[x_1,\ x_2]$, T-delay time, and θ_i-is the angle that defined the position of tooth-*i* at the normal of cutting surface. The tooth-*i* remains in contact with workpiece only for the condition of $\theta_i \in [\theta_{in}, \theta_e]$, where θ_{in} is the angle of entry cutter in material and θ_e is angle of outside cutter from material (Fig.2).

Considering that the value of $k_o^{(a)}$ remained the same for each tooth of milling cutter-*z*, can be written [4-6,19]:

$$\Delta F = \Delta F^{(i)} + \Delta F^{(e)}$$
$$\Delta F = -H(p) \cdot x$$

(8), where the transfer function of milling process is:

$$H(p) = H^{(i)}(p) - \mu e^{-pT} H^{(e)}(p) \quad (9)$$

In final is obtained the relation:

$$\begin{bmatrix} \Delta F_1 \\ \Delta F_2 \end{bmatrix} = \Delta F = -H(p) \cdot x$$

(10), and the expression of H(p) is:

$$H(p) = \left(1-\mu e^{-pT}\right)\begin{bmatrix} \sum_{i=1}^{z} k_i^{(a)}(\theta)\cos(\theta_i+\theta_0)\cos\theta_i, & \sum_{i=1}^{z} k_i^{(a)}(\theta)\cos(\theta_i+\theta_0)\sin\theta_i, \\ \sum_{i=1}^{z} k_i^{(a)}(\theta)\sin(\theta_i+\theta_0)\cos\theta_i, & \sum_{i=1}^{z} k_i^{(a)}(\theta)\sin(\theta_i+\theta_0)\sin\theta_i, \end{bmatrix} \quad (11)$$

In similar mode can be determined the components of dynamic cutting force for bevel milling cutter, where the cutting force has tree components located parallel with an reference tree-orthogonal system, or for general case that the milling cutter is considered as an un-deformed body with six DOF. From Eq.(11) can be observed that the components of dynamic force ΔF_l (l=1,2) depend of simultaneous nr. of teeth engaged in cutting (by θ_{in}-θ_e and pitch of tooth) and θ_0, which can written [4,6,19]:

$\theta_i = 2\pi \cdot n \cdot t + i \cdot \Delta\theta$ (12), where: $\Delta\theta$-is circular pitch of teeth.

For a multivariable DMS [4,6] is consider an particular case when (SE) has the eigenvalues of moving equal with the axes nr. of Cartesian rectangular system:

$$x - \gamma \cdot G(p) \cdot \gamma^{-1} \cdot \Delta F(x, px) = \gamma \cdot G(p) \cdot \gamma^{-1} \cdot F_0 \quad (13),$$

where the characteristic equation is:

$\left|I - \gamma \cdot G(p) \cdot \gamma^{-} \cdot H(p)\right| = 0$ (14), in this case the unit matrix has the same dimension with transfer function of elastic system-G(p), respectively the matrix transfer function of cutting process- H(p).

For the study of (SE) with two DOF for the milling process of cylindrical milling cutter with straight teeth is presented the model from Fig.3 [4,6].

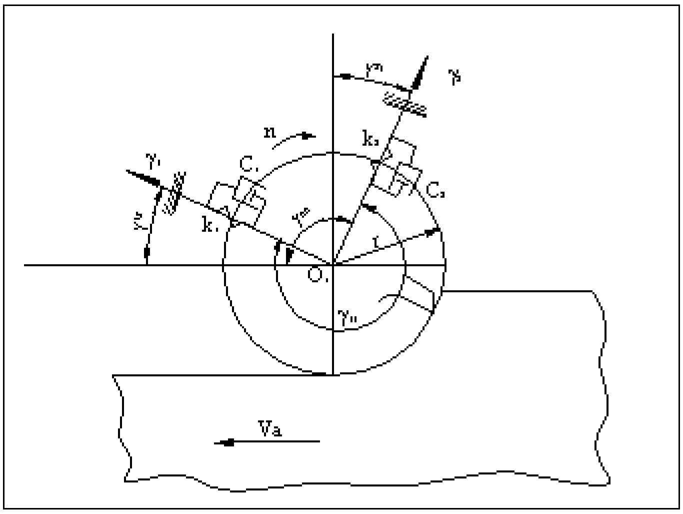

Fig.3. Model of orthogonal milling process with 2 DOF, used milling cutter with straight teeth [6].

From the model of milling process with two DOF and used milling cutter with straight teeth can be written the matrix [4,6,19]:

$$\gamma = \begin{bmatrix} \cos\gamma_{11} & \cos\gamma_{21} \\ \cos\gamma_{12} & \cos\gamma_{22} \end{bmatrix} \tag{15}$$

$$G(p) = \begin{bmatrix} 1/\left(m_1p^2 + c_1p + k_1 + \varphi_{11}\gamma_{01}^2\right) & 0 \\ 0 & 1/\left(m_2p^2 + c_2p + k_2 + \varphi_{12}\gamma_{02}^2\right) \end{bmatrix}$$ (16), where is supposed that associated equivalent weights of eigenvalues of moving (which have non-linear characteristics) are different, and γ_{01} and γ_{02} are get form relation:

$$x_0 \cdot \cos\alpha_{xl} = \sum_{q=1}^{np} \gamma_{0q}\cos\gamma_{ql} \tag{17}$$

By altering the parameter p=jω the Eq.(13) is led to obtain the stability curves of DMS into stability range [4,6]:

$$\begin{aligned} k_0^{(a)} &= k_0^{(a)}(\varepsilon,\omega) \\ f(\varepsilon,\omega) &= 0 \end{aligned} \tag{18}$$

3. MODEL OF SIDE MILLING CUTTER WITH INSERTS

The milling process model of traditional precision milling machine for tool-shop (Fig.4) has done, using side-milling cutters with three cutting edges and eight positive triangles inserts, type TCMT 16T308T N7010 [11,12]. The notation of axes milling machine is as: (X)–axe of horizontal main spindle; (Y)–axe of cross feed table related to main spindle; (Z)–axe of vertical displacement table. The particularity of this machine (FUS22 modified), higher rigidity of X-axe, lead to ignore the force F_x, which doing that only forces of Y and Z-axes exert influence in milling process.

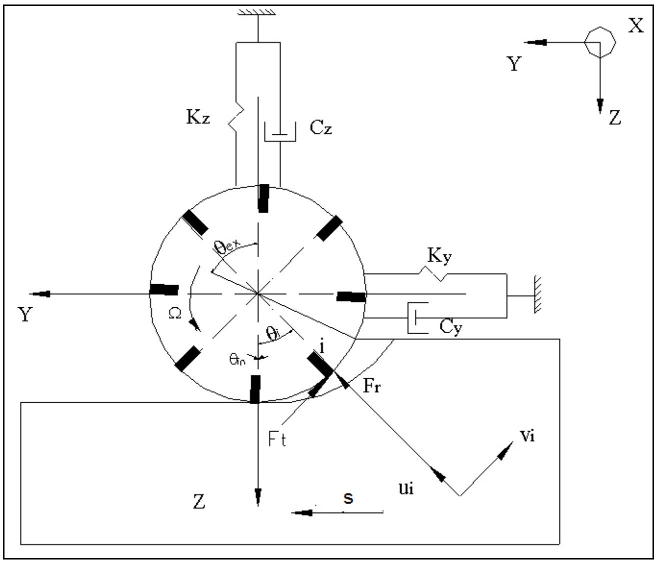

Fig.4. Dynamic model of peripheral milling process [11,12]

The cutting force is determined by its two components (F_t-tangential force, F_r-radial force), which have formula [1,4,11,12,15-17]:

$$F_{ti} = K_t \cdot \bar{t} \cdot g(\theta_i)$$
$$F_{ri} = K_r \cdot F_{ti}$$

(19), where: K_t, K_r–coefficients of cutting force, ($\bar{t}$)-depth of cut, and $g(\theta_i)$- thickness cut. If difference $\theta_{ex} - \theta_{in} > \theta_p$, where θ_p-is pitch of milling cut, that due to more teeth of cutter would be in work and the angle θ_i to became $\theta_i = \theta + i\theta_p$, (θ)-is reference angle tooth for $i = 0$, and $\theta_p = 2\pi/N_z$, where N_z- is inserts number.

After all transformations and calculus, dynamic equation of cutting force in matrix form became:

$$\begin{Bmatrix} F_y \\ F_z \end{Bmatrix} = \frac{1}{2}\bar{t} \cdot K_t \begin{bmatrix} b_{yy} & b_{yz} \\ b_{zy} & b_{zz} \end{bmatrix} \begin{Bmatrix} \Delta_y \\ \Delta_z \end{Bmatrix}$$

(20), where: [B]-is the matrix of directional force process coefficients. The tool vibration assumes to occur at chatter frequency-ω_c, when a marginally stable depth of cut has taken. The forces have described as harmonic functions and dynamic equation of milling cutter has written [1,4,8,11,12]:

$$\{F\}e^{i\omega_c t} = \frac{1}{2}\bar{t} \cdot K_t \left[1 - e^{-i\omega_c T}\right][B_0][\Gamma(i\omega_c)]\{F\}e^{i\omega_c t}$$

(21), where: $[B_0]$-is matrix of average component of Fourier series expansion, $[\Gamma(i\omega_c)]$-matrix of transfer function in tool-workpiece contacted zone. The Eq. (21) has the solutions if its determinant is zero:

$$\det\left[[I] - \frac{1}{2}\bar{t} \cdot K_t \left(1 - e^{-i\omega_c T}\right)[B_0][\Gamma(i\omega_c)]\right] = 0 \qquad (22)$$

The eigenvalues of characteristic equation is writing as:

$$\Pi = -\frac{N_z}{4\pi}\bar{t}\cdot K_t\left(1-e^{-i\omega_c T}\right) \tag{23}$$

$$\det\left[[I]+\Pi\left[\Gamma_0(i\omega_c)\right]\right]=0 \tag{24}$$

The eigenvalues of Eq. (23) can be calculated for a chatter frequency-ω_c, cutting force coefficients are depend by condition of material-geometric tool, penetration angle(θ_{in}, θ_{ex}) and the transfer function of structure $\Gamma_0(i\omega_c)$. After certain simplifications and changed eigenvalues and expressed $e^{-i\omega_c T} = \cos\omega_c T - i\sin\omega_c T$, the critical depth of cut at resonance is:

$$\bar{t}_{cr} = -\frac{2\pi\Pi_R}{N_z K_t}\left[1+\frac{\sin^2\omega_c T}{(1-\cos\omega_c T)^2}\right]$$ (25), where: Π_R-is real part of eigenvalue, ω_c T-delay phase between vibration at successive period of tooth. The modeling of milling process has made for 4 side-milling cutters with inserts, ones of them was the cutter FI160 that is a side-milling cutter with three cutters, Ø160mm and eight interchangeable triangles carbide inserts type TCMT 16T308T N7010 in zigzag position (bevel angle 7^0), with angle $\alpha = +6^0$ and pitch angle $\delta = 45^0$. As entry's data were the values: $\bar{t}_{\max}$ =1mm, θ_{max}=9.07°, resulted K_t=983N/mm^2 and K_r=0.130 (Fig.5).

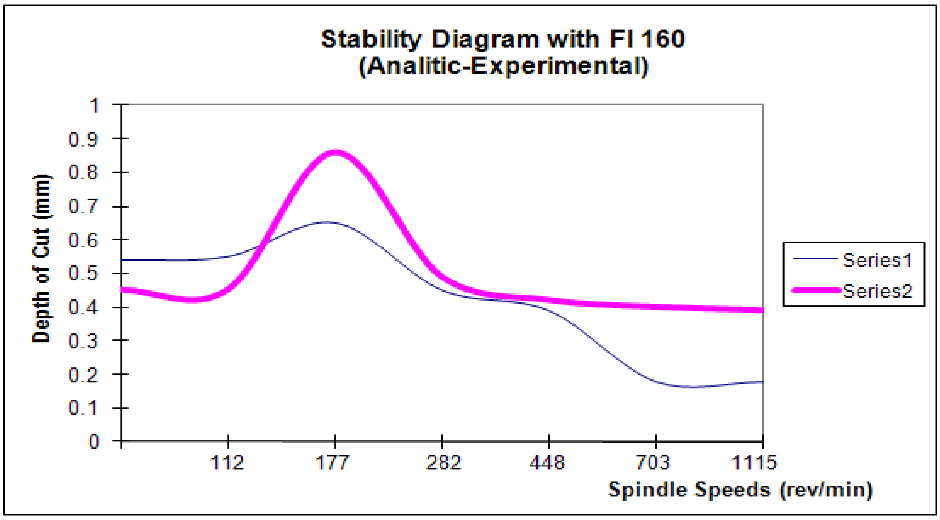

***Fig.5**: Stability diagram of milling process by used cutter FI160 for K_t=983 N/mm^2; Series1-experimental mode, Series2-analytical mode*

By analysing the diagrams of milling process by used milling cutter FI160 a little difference of 0.02-0.07mm, between the analytical and experimental mode can be seen, which confirms the mathematical model of milling process.

4. CONCLUSIONS

This paper has presented certain models of milling process used for assessing of dynamic stability of machine tools. In addition, a model of milling process by used a side-milling cutter with inserts in details is presented.

These mathematical models are demand to determination of stability diagram's cutting process and the investigation of machine tools, assured the possibility to find the solutions for performing the process and improving the performances of machine tools.

5. ACKNOLEDGMENT

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6. REFERENCES

[1] ALTINTAS, Y.: “*Manufacturing Automation: Metal Cutting Mechanics, Machine Tool Vibration and CNC Design*”, Cambridge University Press, ISBN 0-521-65973-6, pp. 33-46, 2000.

[2] ALTINTAS, Y., LEE, P.: “Mechanics and Dynamics of Ball End Milling”, *Transaction of ASME, Journal of Manufacturing Science and Engineering*, Vol.120, pp. 684-692, 1998

[3] ALTINTAS, Y., BUDAK, E.: “Analytical Prediction of Stability Lobes in Milling", *Annals of the CIRP*, Vol.44, No.1, pp. 357-362, 1995

[4] CHIRIACESCU, C.S.: “*Stability in Dynamic of Metal Cutting”,* Elsevier Science Ltd, New York, Amsterdam, 1990

[5] CHIRIACESCU T.S.: *"Vibratiile Masinilor Unelte*", Brasov University Editor, 1984

[6] CHIRIACESCU T.S.: *"Stabilitatea in Dinamica Aschierii Metalelor"*, Academy Ed., Bucharest, 1984

[7] ENDRES, W.J.: “A Quantitative Energy-Based Method for Predicting Stability Limit as a Direct Function of Spindle Speed for High-Speed Machining”, *Transactions of NAMRI/SME*, Vol.24, pp. 27-32, 1996

[8] LANDERS, R.G., USLOY, A.G., FURNESS, R.J.: “Process Monitoring and Control of Machining Operations”, *Mechanical Systems Design Handbook*, O.D.I. Nwokah and Y. Hurmuzlu (eds.), CRC Press, Chapter 6, pp. 85–119, 2002.

[9] LANDERS, R.G., ULSOY, A.G.: “Model-Based Machining Force Control” *ASME Journal of Dynamic System, Measurement and Control*, Vol.122, No.3, pp.521-527, 2000

[10], MINIS, I, YANUSHEVSKY, R.: “A New Theoretical Approach for the Prediction of Machine Tool Chatter in Milling”, *ASME Journal of Engineering for Industry*, Vol.115, No.1, pp. 1-8, 1993.

[11] POP, P.A.: “The Analysis of Dynamic Stability at Milling Machine Tools” ASME International Design Engineering Technical Conference & Computers and Information Engineering Conference IDETC/CIE2009, August 30-September 2, 2009, San Diego, CA, *Proceeding of DETC2009*, Paper No-DETC2009-86070, pp.1-10, Order No. 1823DV, ISBN 978-0-7918-3856-3, ASME International 2009.

[12] POP, P.A.: “The Study Of Cutting Forces About Dynamic Stability Of Milling Machine Tools”, The 9th Biennial ASME Conference on Engineering Systems Design and Analysis ESDA2008 July 7-9, 2008, Haifa, Israel, *Proceedings of ESDA2008*, Paper No. ESDA2008-59231, pp.1-10, Order No.1794CD, ISBN 0-7918-3827-7, ASME International 2008

[13] POP, P.A., BEJINARU-MIHOC, G.: “Proceedings Used For Enhancing Machining Process”, 11th International Research/Expert Conference ”Trends in the Development of Machinery and Associated Technology”, *Proceedings of TMT 2007*, Hammamet, Tunisia, 5-9, Sept. 2007, Vol. XXVIII(62), pp. 191-194, ISBN 978-9958-617-34-8, 2007.

[14] POP, P.A., UNGUR, P., BEJINARU-MIHOC, G.: “Investigation of Dynamic Machining Process at Machine Tools”, 3th International Conference of European Tradition and Influence in Engineering Creation MSE 2007, Sibiu, July 12-14 2007, *MSE2007 Proceedings Sibiu-Extended Abstracts*, pp.61-62, “Lucian Blaga” Univ. of Sibiu Ed., ISSN 1843-2522. Academic *Journal of Manufacturing Engineering*, Vol.5, Supplement No.2/2007, pp.104-108, Polytechnic Univ. of Timisoara Ed. ISSN-1583-7904, 2007

[15] POP, P.A.: “Improving the Milling Process by Reduce the Milling Cutting Forces”, Proceedings of the International Conference of Manufacturing Systems ICMaS 2004-Bucharest, Univ. Polytechnic of Bucharest 7-8 Oct.2004, *Romanian Journal of Technical Sciences, Applied Mechanics*, Tome 49, Special Number 2004, pp.301-304, Romanian Academy of Publishing House, ISSN 0035-4074, 2004

[16] POP, P.A.: “The Diagnosis of Milling Process Using the Side Milling Cutter with Three Cutting Edges”, The 9th International Scientific Symposium “Quality and Reliability of Machines”, International Machinery Faire, 25-26 May Nitra 2004, ISBN 80-8069-369-2, pp. 375-377, Nitra, Slovakia, 2004.

[17] POP, P.A.: “Determination of the Milling Cutting Forces”, The 9th International Scientific Symposium “Quality and Reliability of Machines”, International Machinery Faire, 25-26 May Nitra 2004, ISBN 80-8069-369-2, pp. 378-380, Nitra, Slovakia, 2004.

[18] POP, P.A.: "Analiza vibraţiilor procesului de aşchiere prin frezare care conţin forţe neliniare", Proceeding of the X-th International Conference on Mechanical Vibrations, *Journal of Politehnica University Timisoara*, Mechanical Vibration Tom 47/61, vol. 1, ISSN 1224-6077, pp. 317-322, 2002.

[19]. POP, P.A.: *"Modele matematice utilizate în analiza stabilităţii dinamice a procesului de* prelucrare prin frezare", Proceeding of the X-th International Conference on Mechanical Vibrations, Journal of Politehnica Univ Timisoara, Mechanical Vibration Tom 47/61, Vol 1, ISSN 1224-6077, pp. 263-270, 2002

[20] RADULESCU, R., KAPOOR, S.G., DEVOR, R.E.: "An Investigation of Variable Spindle Speed Face Milling for Tool-Work Structures with Complex Dynamics, Part 1: Simulation Results and Part 2: Physical Explanation", *Journal of Manufacturing Science and Engineering*, Vol.119, No.3, pp.266-280, 1997.

[21] SHRIDAR, R., HOHN, R.E., LANG, G.W.: "A Stability Algorithm for the General Milling Process: Contributions to Machine Tool Chatter Research-7", *ASME Journal of Engineering and Industry*, Vol.90, No.2, pp.330-334, 1968.

[22] SMITH, S., TLUSTY, J.: "Efficient Simulation Programs for Chatter in Milling", *Annals of CIRP*, Vol.42, No.1, pp. 463-466, 1993.

[23] TARNG, Y.S., YOUNG, H.T., LEE, B.Y.: "Analytical Model of Chatter Vibration in Metal Cutting", *International Journal of Machine Tools and Manufacture*, Vol.34, No.2, pp. 183-197, 1994

[24] TLUSTY, J.: "Dynamics and High Speed Milling", *Journal of Engineering for Industry*, Vol.108, No.2, pp. 59-67, 1986

[25] TLUSTY, J. ISMAIL, F.: "Special Aspects of Chatter Milling", *ASME Journal of Vibration, Acoustics, Stress and Reliability in Design,* Vol. 105, No.1, pp. 24-32, 1983

[26] TLUSTY, J., ISMAIL, F.: "Basic Nonlinearity in Machining Chatter", *Annals of CIRP*, Vol.30/1, pp. 299-304, 1981

[27] TSAI, M.D., TAKATA, S., INUI, M., KIMURA, F., SATA, T.: "Prediction of Chatter Vibration by Means of a Model Based Cutting Simulation System", *Annals of CIRP*, Vol.39. No.1, pp. 447-450, 1990

[28] WECK, M., ALTINTAS, Y., BEER, C.: "CAD Assisted Chatter-Free NC Tool Path Generation in Milling", *International Journal of Machine Tools and Manufacture*, Vol.34, No.6, pp. 879-891, 1994.